MODERN OBSERVATIONS USING THE 1702 MERIDIAN LINE OF THE BASILICA OF SANTA MARIA DEGLI ANGELI E DEI MARTIRI (ROME)

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This article is based on the talk given by the first-named author at the 2016 BSS Conference in Liverpool.

The giant meridian lines that can still be found in a handful of major cathedrals in Italy and France have long fascinated the first author. At solar noon an aperture high in the south wall of each church admits a spot of sunlight that crosses the meridian line at different locations dependent on the season. In the Spring term of 2015 the opportunity arose to lead (together with art historian Lane Eagles) a program of studies in Rome for seventeen undergraduates of the University of Washington, Seattle. Through lectures, readings, and visits to field sites in and near Rome, we studied the many interactions among art, science and religion over the centuries. Students also learned about sundials through constructing and using their own personal dials.

In particular we studied the history, art, architecture and astronomy of Santa Maria degli Angeli e dei Martiri (latitude 41° 54′ 11″ N, longitude 12° 29′ 51″ E), a unique basilica that arguably has the finest extant meridian line of them all.[1] In this church each student observed one or more solar noons over the term. Once back in Seattle, the authors analysed these and other observations in order to determine accurately the obliquity of the ecliptic (the 'tilt' of the Earth's axis) and the expected difference from values measured three centuries earlier. Also determined were accurate values for the length of the year and the time of vernal equinox, historically important for determining the date of Easter.

Historical Setting

Giant meridian lines

During the 17th and 18th centuries many meridian lines were constructed in churches, palaces, mansions and astronomical observatories.

Constructing them is straightforward in principle. A small aperture high in a roughly south-facing wall or in a ceiling admits a spot of sunlight. When the sun is highest in the sky each day, at solar noon, mark on the floor where the solar image falls.[2] As the seasons pass and the sun moves north and south in the sky, the series of marks will describe a north–south line, each point on the line corresponding to two dates of the year, one between the winter and summer solstices as the sun moves north, and the other during the following six months. Next make the north–south line permanent by laying a brass strip in the floor and label it with signs of the zodiac as well as special calendar dates.[3] The line can be further marked with more scientific quantities such as the sun's altitude angle and declination, or a linear scale in metres or other units. Now in successive years one can determine calendrical information from *where* on the line the solar image crosses, as well as the precise time of solar noon by *when* it crosses.

The motivation to build large meridian lines in churches was first to establish calendrical quantities such as the length of the year and the time of vernal equinox, both vital to the Church for establishing the changeable date of Easter and other religious holy days.[4] But at the same time, one could study more subtle effects such as atmospheric refraction, or the obliquity of the ecliptic, which was suspected to be slowly changing (see the Analysis section). In addition, the *time* of transit provided an accurate measure of the equation of time and could be used daily to set local civil time, as well as the times for services.

Although most meridian lines were never used for any scientific work, created rather for prestige or controlling civil time, others contributed significantly to the astronomical knowledge of the day. The 1999 book *The Sun in the Church* by John Heilbron is the definitive historical study of meridian lines in churches.[5] The first one of note was built by Paolo Toscanelli (1397–1482) in 1468 for the Cathedral of Florence. The location of its aperture, fully 90 metres up in the famous dome, means that the solar image reaches the floor only near the summer solstice. The most scientifically fruitful was built in the Bologna Basilica of San Petronio in 1655 by the astronomer Giovanni Cassini (1625–1712), before he moved to Paris for the bulk of his career. Another notable extant meridian line is that of St Sulpice church in Paris, built in 1743 by Pierre Le Monnier (1675–1757). Though simple in design, the scale, precision, and elaborate artwork of these meridian lines reveal the admiration church officials had for the science of their time.

Fig. 1. Looking towards the northwest transept of Santa Maria degli Angeli e dei Martiri, Rome. The 30-metre high vaulted ceiling is a remarkable remnant of the original Diocletian baths of AD 306. Photo: Sullivan.

Santa Maria degli Angeli e dei Martiri

The beginnings of Santa Maria degli Angeli e dei Martiri in Rome are either AD 306 or 1561, depending on one's point of view. At the latter time Pope Pius IV (reigned 1559–65) authorized the construction of a church on the site of the ruins of the ancient huge baths (~300 metres on a side!) built by the Emperor Diocletian and dedicated in 306. The repurposing of this site expressly illustrated the triumph of Christianity over Roman paganism – the first Emperor to convert to Christianity was Constantine, only two decades after Diocletian. The name of the basilica, referring to 'martyrs', was to honour the many slaves who died building the baths.

The Pope hired none other than Michelangelo (1475–1564), then aged 86, to design the basilica. Ever the innovator, he incorporated surviving portions of the ancient baths into his design. The result was that 30-metre high vaults and huge hallways (only a small portion of the former baths) were transformed into a majestic nave and transept (Fig. 1).[6] He also chose to make the church's exterior the most radical in Rome by keeping it unadorned and looking like an ancient ruin, as it still does today.[7]

Bianchini's meridian line

A century after the Gregorian calendar reform of 1584, various experts felt that it still needed fine-tuning. Pope Clement XI (reigned 1700–21) thought that his prestige could be enhanced if Rome acquired its own large meridian line. He thus commissioned the polymath Francesco Bianchini (1662–1729), a sometime colleague of Cassini's, to build one in whichever Roman church he found most suitable. Bianchini (Fig. 2) considered even St Peter's, but its scale was too large. He finally chose Santa Maria degli Angeli because its geometry worked and its 1400-year history guaranteed that its walls had completely settled.

Fig. 2. Francesco Bianchini (1662–1729).[11]

Fig. 3. Looking south along about two-thirds of the length of the meridian line. The southern aperture (close-up in Fig. 5) can be seen as a dark feature cut through the architrave, to the lower right of the windows at the top; its height is 20.3 metres above the floor. Photo: Sullivan.

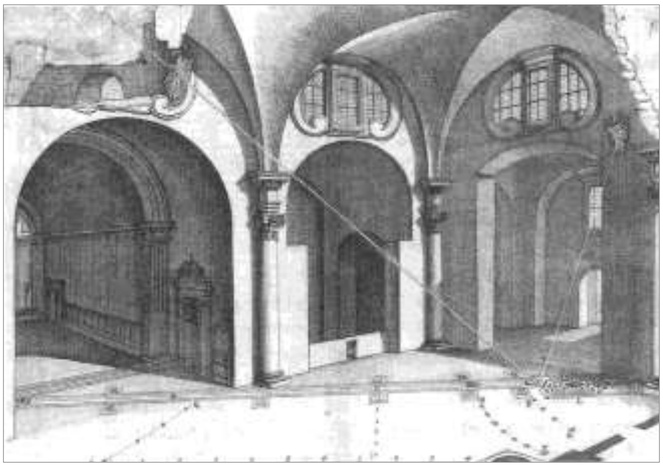

Fig. 4. Schematic diagram of the basilica, showing the light paths from both the southern aperture (to the right) and the northern (Polaris) aperture.[10]

Fig. 5. The southern aperture. A major revision of the basilica interior in the mid-18th century introduced an architrave, complete with its accommodating cutout section. Light from the sky can just be seen coming through the aperture (diameter about 2–4 cm). Photo: Sullivan.

Over a remarkably short time in 1702 Bianchini built what many contemporaries hailed as the most beautiful, useful and interesting meridian line in Europe (Figs 3 and 4). The aperture[8] is at a height of 20.344 metres on the south wall (Fig. 5), which then means that the brass meridian line (of width 3.4 cm within a broad marble band) has a summer-to-winter length of 37 metres as it slices across the transept. Fig. 6 shows the basic geometry of how the solar image falls on the meridian line over the year, the huge scale of this scientific instrument allowing a very accurate measure of solar declination. The meridian line is not

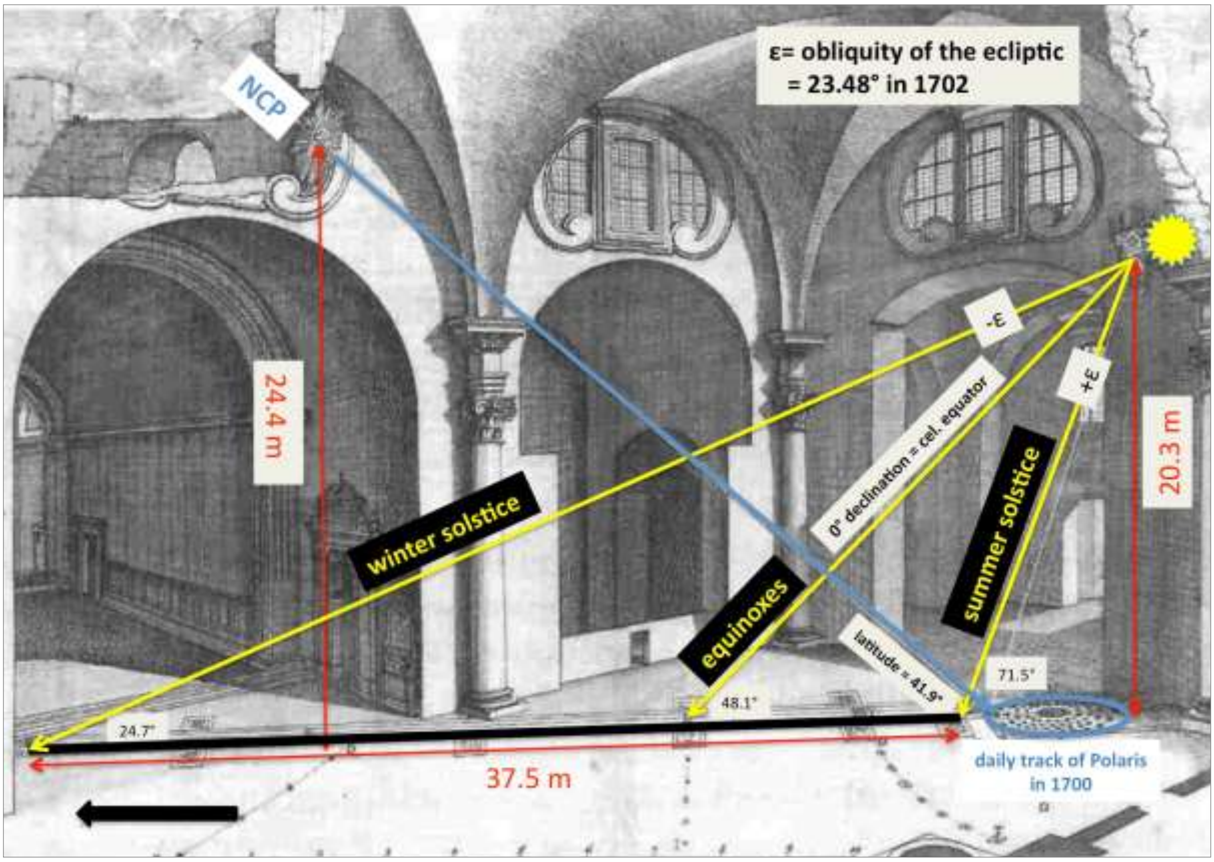

Fig. 6. Explanatory features (overlaid on Fig. 4) illustrating the usage of the two apertures and the annual change in declination of the sun. The North Celestial Pole (NCP) aperture allowed light from Polaris (shown as a blue line) to describe a daily ellipse on the floor as indicated.

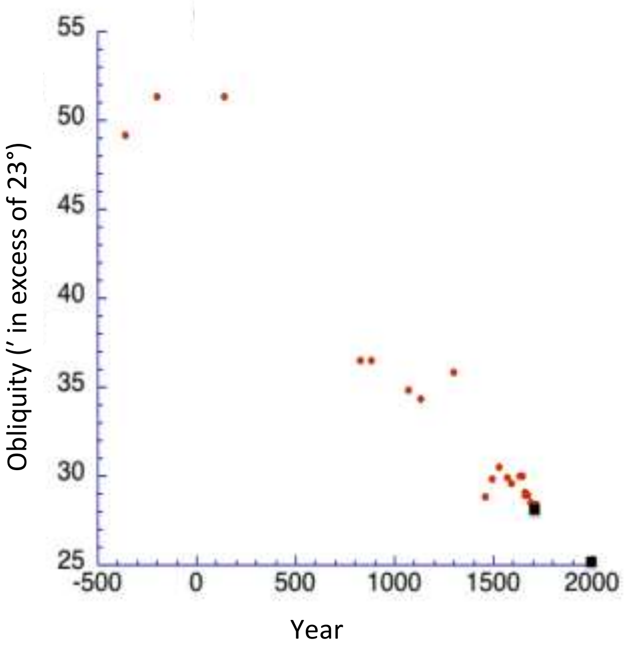

Fig. 7. Historical measurements of the obliquity of the ecliptic, as known circa 1700. The black squares refer to Bianchini's 1703 value and the result of the present study. Plot based on tables in Heilbron.[5]

sequentially labelled with the dates of the year, but does feature ornate panels depicting the constellations of the zodiac, two scales related to the angle of the noontime sun from the zenith, and a half-dozen plaques commemorating various events, each at the correct position for the date of the event. The earliest is dated 20 August 1702, when the Pope himself came to inspect his new meridian line.

Besides the meridian line intended for solar observations, Bianchini added several unique features. The first was the ability to set up a small refractor telescope along the line, in order to measure accurately the altitude angle and transit times not only of the sun, but also of stars and planets. Bianchini could observe the brightest stars even in the daytime to determine accurately their celestial positions with respect to the sun.[9]

The movable small telescope could also observe *to the north*, specifically the star Polaris, through a second high aperture facing north (Fig. 6). This enabled a measurement of latitude, and was the inspiration for a unique set of nested ellipses marked on the floor near the southern end of the meridian line (Fig. 18; see the section on Polaris).

Over the first 15 months of the meridian line's existence (September 1702 to December 1703), Bianchini observed everything he possibly could: 53 days of solar transits, bright stars such as Sirius and Arcturus, planets, Jupiter's moons, eclipses of the moon (but not on the meridian), and Polaris to the north. He quickly published these observations in a book dedicated to the Pope,[10] concluding that the best overall value for the length of the year (as determined much earlier by others) was in accord with that adopted by the Gregorian reform, as well as obtaining a precise value

Fig. 8. The students and instructors of the Program in the Spring of 2015, posing in front of St Peter's dome while visiting the Tower of Winds in the Vatican. Photo: Tour guide.

for the obliquity of the ecliptic. He and others continued to use the meridian line for much of the 18th century.[11]

The obliquity of the ecliptic — does it change over time?

Astronomers in the 17th and 18th centuries argued about whether or not the obliquity of the ecliptic ε, the 'twenty-three and a half degree' tilt of the Earth's axis with respect to the plane of the Earth's orbit, was in fact changing over the centuries. If indeed it was changing, what was causing the change, and when might it stop, or reverse its course? The data in hand were as shown in Fig. 7. One had three groups of values: ancient authorities such as Ptolemy at roughly 23.8°, Islamic astronomers ca. AD 1000 at ~23.6°, and more accurate and numerous recent measures at 23.5°. At face value it looked like a significant decline, but how much stock should one put into the comparatively crude pre-telescopic values? And why were late 17th century astronomers disagreeing amongst themselves as to the correct contemporary value? Some even trusted meridian lines more than the telescopes of the day. Not until the end of the 18th century did a consensus develop that the decline was real, that it was due to the gravitational effects of Jupiter and Saturn on the bulge of the Earth's equator, and that its value was 47″ per century.[12,13]

One key motivation for our own observations at Santa Maria degli Angeli was to see whether or not we could determine the early 21st century value of ε with sufficient accuracy actually to detect its decline since the time of Bianchini three centuries before. We of course knew that the decline has been 2.4′, but did not know beforehand whether or not we could detect so small a change.

Our Observations

During the Spring of 2015 our class (Fig. 8) observed as many transits of the solar image at Santa Maria degli Angeli as schedules and weather would allow. An

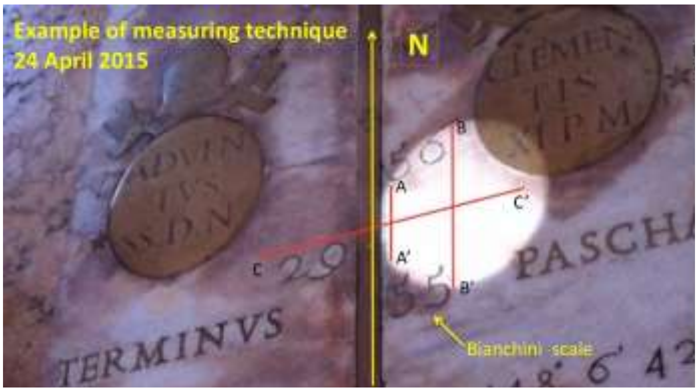

Fig. 9. A frame from our movie of 24 April 2015 to illustrate the image analysis technique for solar image position. During our Spring term the solar image typically moved with a speed of ~9 cm/min and had a width of ~20 cm. The two scales of Bianchini can be seen: on the left '29' refers to a solar zenith angle of 29° and on the right '55' refers to tan (zenith angle) = 0.55. The entire 3-minute movie can be found by searching within YouTube for 'obliquity data' (including the inverted commas).

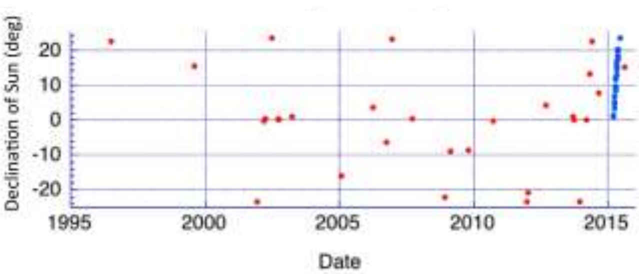

Fig. 10. The 54 observations in our dataset, scattered over the period 1996–2015: red points are taken from images by others and blue are from our campaign in Spring 2015.

Fig. 11. Examples of other images used in our study. Note the different sizes and shapes and angles of view. The image size at the winter solstice is ~110 × 45 cm. Credits: 21 December 2013 and 25 April 2014 (C. Sigismondi, YouTube); 4 June 2014 (anonymous); 20 April 2015 (a frame from one of our movies).

observation consisted of a 2–3 minute movie at solar noon of the moving solar image. Fig. 9 shows a frame from a typical example of these movies; the full movie can be found by searching within YouTube for 'obliquity data' (including the inverted commas). In the end we gathered 23 days of usable data over the period 23 March to 19 June 2015.

In order to supplement our springtime dataset, we scoured publications and the Internet for any usable still images or movies of meridian solar images. These images were of variable quality, but usually satisfactory. Altogether this yielded 31 more observations, for a total of 54 over the span 1996 to 2015 (Fig. 10).[14]

For a given day we measured the declination of the sun by the position on the meridian line where the solar image crossed.[15] Because the images are taken from a wide variety of angles with respect to the meridian line, our usual method of image analysis used the facts that the spot of light (a) is elliptically shaped,[16] and (b) moves perpendicular to the meridian line at solar noon.[17] In Fig. 9 lines AA′ and BB′ span the solar image in two different places and are constructed parallel to the meridian line.[18] The line CC′, the perpendicular bisector of both AA′ and BB′, then intersects the meridian line at the desired location. In practice this was done for two images (when available) on each day, on either side of the meridian line; the two results were then averaged. Fig. 11 gives examples of images taken by others at different locations on the meridian line, on various dates from a variety of angles; note the different shapes and brightnesses of the images.

Bianchini supplied us with a linear scale all along the meridian line[19] and with our method we estimate that we could measure a solar image's centre to ±4 mm, which corresponds to an error of ~ ±0.6′ at the summer solstice, decreasing to ~ ±0.1′ at the winter solstice. The measured solar declination for a given day was then found from the algorithm shown in Fig. 12. Note that we had to correct (as

A Sample Datum: 24 April 2015

Basic equation on the meridian:

declination of sun = latitude minus solar zenith angle

latitude of Santa Maria degli Angeli = 41.903°

measured transit position on the line: 55.60 on Bianchini's scale
tan (zenith angle) = 0.5560
solar zenith angle = 29.074°
refraction correction to zenith angle = +0.0087°

measured declination of sun = +12.820°

compare with known value = +12.837°
difference = 0.017° = 1.0′

Fig. 12. The algorithm for determining solar declination from a measurement on the meridian line, starting with data from Fig. 9.

did Bianchini, using values measured by Cassini) for the fact that light is bent downwards as it refracts through the Earth's atmosphere. This means that any object viewed through the atmosphere appears higher in the sky (larger altitude angle) than it really is. In Rome the noontime sun altitude varies from 25° in the winter to 72° in the summer, which means that the solar image is 'lifted' by amounts between 2.2′ (winter) and 0.4′ (summer). Although the exact amount of refraction depends on temperature and pressure, adopting standard values of 20°C and 1.0 atm was adequate given other errors in our data.

Fig. 13 plots the difference for each day between our measured solar declination and the known value for the same day. The standard deviation of the dataset is 0.9′, which provides a good estimate of how well we were able to measure declinations. This error arises from many possible causes, but only about one-half of it is due to the determination of the centre of the solar image. The remainder is probably due to the actual geometry of the aperture/wall/floor/meridian line differing from what we assumed. In addition, there are unfortunately no 'tick marks' on the line, only the large, ornate numerals alongside (Fig. 9). We chose the *centre* of each numeral to be the exact location to which it referred, but there could well be individual or systematic errors in where these numerals are placed.[20]

Sigismondi has investigated many properties of the meridian line;[21] for instance, he finds that its direction deviates 4.5′ to the east of north. Although important for any timing observations, this has minimal influence on the north–south locations of transit positions. More problematic is that over the past two decades the exact position of the aperture has unfortunately been slightly changed by various persons! An examination of our residuals (measured declination minus correct declination) as a function of time over the years, however, shows no detectable correlation with the history of these changes. In the end, as often with experiments, all we can say is that there exist various known and unknown causes for our measurement errors.

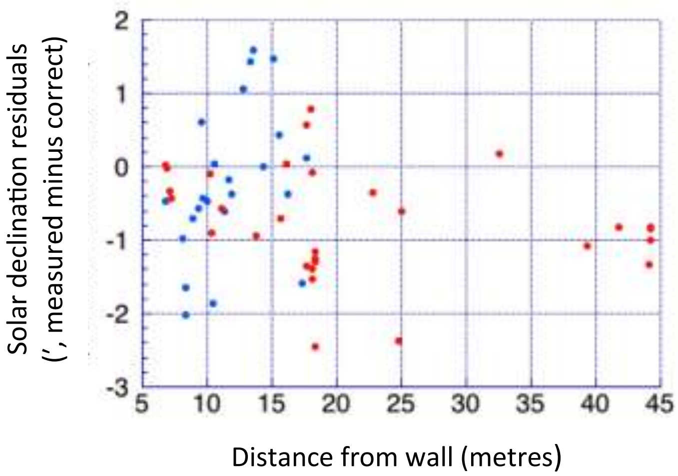

Fig. 13. The difference between our determination of solar declination and almanac values, versus distance from the wall along the meridian. The leftmost data correspond to summer solstice at the southern end of the meridian line and the rightmost winter solstice at the northern end; the equinoxes occur at a distance of 18.2 metres from the wall. Red points are taken from images by others and blue are from our campaign in Spring 2015.

Analysis

Every year the solar declination has a *roughly* sinusoidal variation with period of one year, starting at the vernal equinox, and with an amplitude for the curve equal to the obliquity of the ecliptic. The actual mathematical formulation for the annual variation, however, is complex. The formulation accounts for the obliquity, the elliptical shape of the Earth's orbit, and the Earth's variable speed along that orbit through the seasons. A series of six trigonometric equations allows one to calculate, for any specified date, a predicted declination of the sun to an accuracy of much better than 1′.[22]

Having this mathematical form of how declination through a year varies (our *model*), we then asked the question: for our set of 54 measured declinations over a period of 19 years, which 'best-fit' values of the following three quantities minimize the differences between the model and the data?

P the mean length of the (tropical) year (from one vernal equinox to the next)

T_{VE} the exact time of the vernal equinox (when the solar declination = 0.00°)

ε the obliquity of the ecliptic

A statistical technique called nonlinear least squares (specifically the Levenberg–Marquardt algorithm) allows one to solve simultaneously and optimally for the above three quantities even for a dataset such as ours with very non-uniform intervals between observations. Fig. 14 shows

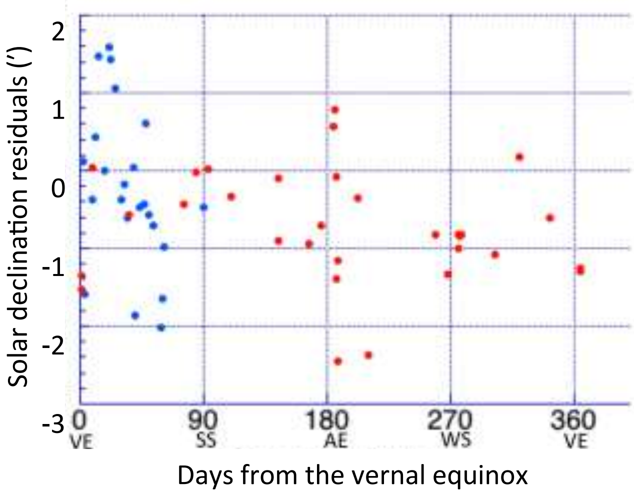

Fig. 14. The difference between our measurements of solar declination and the declinations of the best-fit model versus days from equinox. The equinoxes and solstices are indicated. Red points are taken from images by others and blue are from our campaign in Spring 2015.

the residuals between our best-fit model values of declination and the 54 measured data points. The error estimate (standard deviation) of our best fit model is ±1.1′.

Our best-fit solution is:[23]

P 365.2421832 days = $365^d\ 5^h\ 48^m\ 56^s \pm 136^s$, which is 11 seconds longer than today's accepted value (and 5 seconds less than Bianchini's adopted value)

T_{VE} $1^s \pm 28^s$ earlier on average than the accepted values for each of the years

ε 23.42274° = 23° 25.36′ ±0.26′, which is 0.9′ smaller than the accepted value for the current epoch

Using our best-fit model, all of the 1996–2015 years are folded together in Fig. 15 over one year starting at the vernal equinox; the discrepancies (of typical size ±1′) between the model and the data points are much smaller than the size of the dots in the plot.

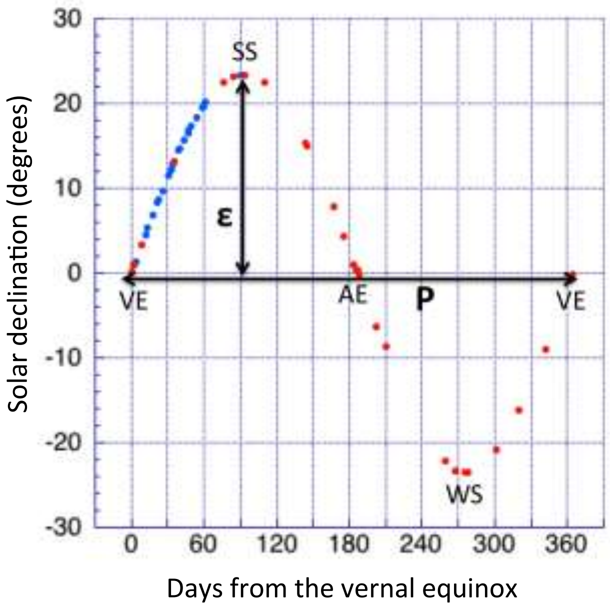

Fig. 15. Solar declinations versus days from equinox (both best-fit values), starting at the vernal equinox. The 54 data points over 19 years have been folded into a single year according to the best-fit value for the length of the year (the time span from one vernal equinox to the next, shown as P). The amplitude of the quasi-sinusoidal curve is the obliquity of the ecliptic, ε. Red points are taken from images by others and blue are from our campaign in Spring 2015.

Bianchini's value for ε in 1702–03 was ε = 23° 28.58′.[24] In the three centuries since the meridian line was built, modern astronomy tells us that the value of ε has declined by 2.40′, whereas our analysis yields a decline of 3.2′ ± 0.3′ from Bianchini's time (Fig. 16). The fact that our derived value for the decline differs from the known value by considerably more than our formal estimate of random error indicates a systematic bias of some sort. Once again, small errors in the assumed church geometry, or in placement of numerals along the meridian line, as discussed in the previous section, would be sufficient.

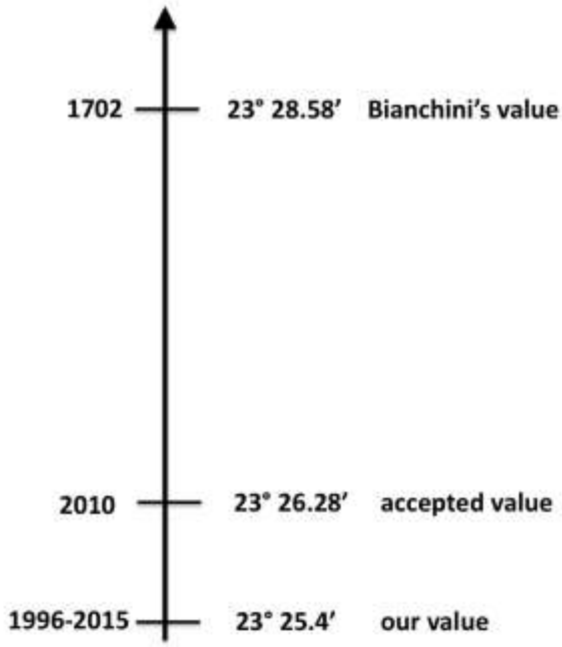

Fig. 16. Our measured value for the obliquity of the ecliptic ε, compared with the accepted value of 2010 (representative of our dataset) and Bianchini's larger value of 1702. Compare Fig. 7.

Nevertheless, we have indeed successfully demonstrated that ε is considerably smaller today than when the Santa Maria degli Angeli meridian line was constructed.

Note that our methodology of measuring ε is distinctly different from that of Bianchini (and all others), who focused on measuring solely the extreme declination values at the winter and summer solstices.[25] Our procedure, using mathematical techniques invented long after Bianchini's era, employs samples taken at random times throughout the seasons as the solar declination varied annually over two decades. Earlier determinations of ε in effect measured only the maximum and/or minimum of the curve shown in Fig. 15 and did not constrain at all the path of the sun during the remainder of the year. In contrast, our method,

Fig. 17. The aperture for the North Celestial Pole and Polaris. The centre of the vertical slot is about 24.4 metres above the basilica floor and 0.91 metres to the west of the meridian line. See Fig. 6. Photo: Catamo and Lucarini.[1]

Fig. 18. *The seventeen nested precession ellipses located near the southern end of the meridian line. Each ellipse is dated with Jubilee Years, including AD 1700/2500 (the outermost, with dimensions of 4.4 × 3.0 metres) and AD 2100 (the innermost). The coat of arms of Pope Clement XI is in the centre, including a large star located at the projection of the North Celestial Pole through the northern aperture. Photo: Catamo and Lucarini.*[1]

although its individual observations are considerably less accurate than typical solstice observations, is based on knowledge of the mathematical form of the entire curve. The two approaches are nicely complementary.[26]

Polaris and the Precession Ellipses

As mentioned before, a unique second aperture was also provided so that observations to the north were possible. An aperture (Fig. 17) high above the floor (24.39 metres) was mounted such that the north star Polaris could be observed with a small telescope near the southern end of the meridian line and displaced 0.91 metres to the west (Figs 6, 18 and 19). Bianchini used this to determine his latitude, and also argued (somewhat disingenuously) that this northern aperture allowed him to determine the precise time of midnight, needed for the timing of various services. We could not observe Polaris *à la* Bianchini because the window behind the aperture is unfortunately now blocked.

Fig. 19. Measuring the dimensions of the precession ellipses (looking north). Note that their major axes run 0.91 metres to the west of the meridian line. Photo: Sullivan.

Instead we turned our attention to the related and unique set of nested ellipses located near the southern end of the meridian line (Fig. 18).[27] Although these ellipses were never historically used for any observations, their design is fascinating.

In Bianchini's time Polaris was located 2.30° from the North Celestial Pole (NCP), in other words its north polar distance (NPD) was 2.30°. Each (sidereal) day Polaris thus described a circle on the sky centred on the NCP and of radius 2.30°. But the NCP, which is the extension of the Earth's rotation axis, does *not* remain fixed with respect to the stars but, rather, every 26,000 years describes on the celestial sphere a wide, roughly circular path whose radius is equal to ε, the obliquity of the ecliptic. This phenomenon is called the precession of the equinoxes and has been well known since ancient times.[28] Precession means that the position of the NCP steadily moves through the heavens. Since Bianchini's time the path of the NCP (Fig. 20) has brought it (by chance) ever closer to the star we call Polaris. Its NPD is today 0.68°, in the year 2100 it will be at its closest (NPD = 0.45°), and thereafter will increase for many millennia.

Bianchini did not make any contribution to the study of precession, but he cleverly used the phenomenon to illustrate the power of astronomy and the longevity of the Church. The ellipses of Fig. 18 represent the projections on the floor of the daily circle of Polaris as they would be observed through the northern aperture at 25-year intervals from AD 1700 to 2500! These dates correspond to 33 of

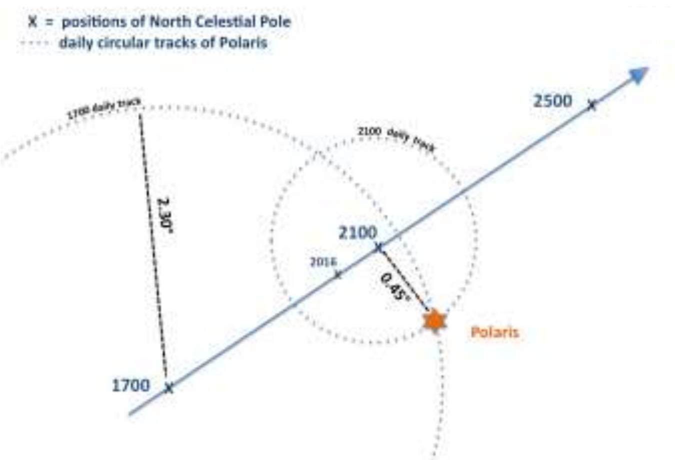

Fig. 20. The phenomenon of precession. A short portion of the precessing path of the North Celestial Pole (NCP) is shown, extending over AD 1700 to 2500, the period covered by Bianchini's ellipses. The daily circuit of Polaris about the NCP is shown at its largest (1700) and smallest (2100). Diagram not to scale.

the celebratory Jubilee Years in the Roman Catholic calendar.[29] Bianchini, however, was able to get by with only 17 ellipses because of the good fortune that the NCP will be closest to Polaris in a Jubilee Year, namely 2100. Then, assuming symmetry between the 400 years before 2100 and those afterwards, he labelled each ellipse with *two* years equidistant from 2100: for example, 1700/2500, 1725/2475, 1750/2450…2075/2125, and finally 2100 by itself.

We made detailed measurements of the dimensions of the nested ellipses, including the 17 major and minor axes (Fig. 19). The axes of the largest (1700/2500) and smallest ellipses (2100) are 4.406 × 2.976 metres and 0.897 × 0.563 metres. The geometry shown in Fig. 21 then allowed us to calculate the values of Polaris's NPD that Bianchini

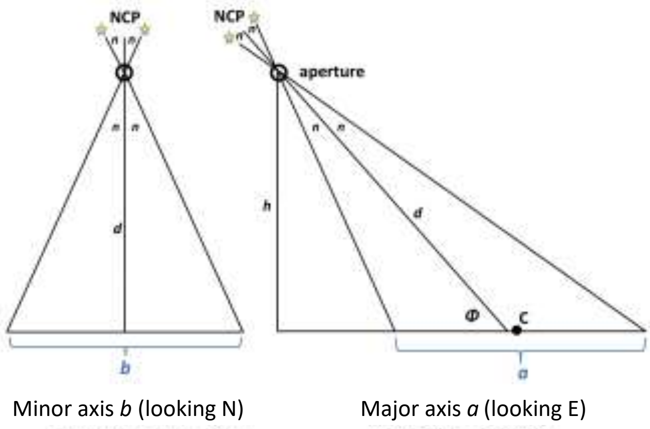

Fig. 21. The geometry of the major axis a and minor axis b of each precession ellipse on the floor, as determined by Polaris's north polar distance (n) (the radius of Polaris's daily path), the height of the aperture h, and the latitude ϕ. C is the centre of the precession ellipse, which is offset from the projection of the North Celestial Pole (end of line d).

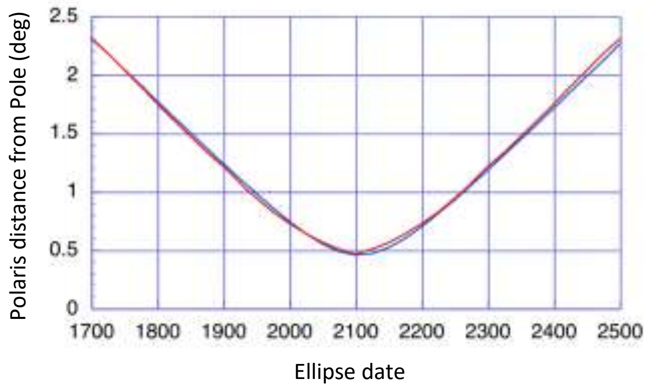

Fig. 22. The varying distance of Polaris from the North Celestial Pole over the period covered by Bianchini's ellipses. The blue line shows modern values and the red line Bianchini's values as deduced from the ellipses of Fig. 18.

apparently predicted and used to draw his ellipses.[30] In Fig. 22 the curve defined by these 33 values is compared with modern precession theory. Bianchini's values of NPD are always within 0.05° (corresponding to ~1 cm error for a typical ellipse axis), but steadily diverge with time, primarily because he did not account for the very effect discussed at length in earlier Sections, namely the changing value of ε, which also affects precession.

We also checked the shape of the largest ellipse in detail and found it to be slightly non-elliptical in the sense that, for its axis ratio,[31] it bulges out too much (by ~1.5%) at locations intermediate to the two axes. Bianchini's design incorporates another interesting geometrical effect: the centre of each projected ellipse does not coincide with the projected NCP (indicated by the large star in Fig. 18; also see Fig. 21). The ellipse centres are not marked on the floor, but we determined them from our axis measurements. We find that our determined centres for successively larger ellipses indeed progressively shift southward away from the NCP point, as they should, but the amount of shift is too small. For example, his offset for the largest ellipse is only 9 cm *versus* the correct 39 cm; perhaps he made the shift smaller for aesthetic reasons, i.e., to keep the nested ellipses more closely centred on the star.[32]

Closing Remarks

There exists a surprising abundance of astronomical phenomena that can be revealed by careful consideration of sunlight passing through a hole in a wall and falling upon a long calibrated line. In the present study it has been possible, with a mathematical analysis of 54 measured solar declinations spread over the seasons and over two decades, to confirm that the obliquity of the ecliptic has indeed grown smaller since Bianchini determined his value in 1702 (Fig. 16). Our dataset has further allowed accurate determinations of the length of the year and the time of vernal equinox. The accuracy of the precession ellipses associated with the meridian line has also been studied for the first time. Notwithstanding some difficulties, these observations are further proof that, after three centuries, the marvellous Santa Maria degli Angeli meridian line remains a remarkably accurate scientific instrument.

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We thank those who supplied the raw solar images for our study, namely (1) the University of Washington (UW) students who participated in the Spring 2015 Program 'Science, Religion and Art in Rome over Two Millennia', and (2) the many persons whose movies and images are posted on the Internet or published.[33] John Heilbron, Frank King and Costantino Sigismondi made valuable reviews of an earlier version of this article. Vital information about the meridian line and its history was provided by Sigismondi and Mario Catamo. Lane Eagles, the co-instructor of the Program, was very supportive of the project, as was the Director of the UW Rome Center, Sheryl Brandalik. The Rev. Don Franco Cutrone's cooperation allowed out-of-hours access to the basilica. Finally, Eric Agol's expertise and software (normally used to hunt for periodicities to discover exoplanets) were essential for the mathematical analysis.

REFERENCES and NOTES

1. M. Catamo and C. Lucarini: *Il Cielo in Basilica: La Meridiana della Basilica di Santa Maria degli Angeli e dei Martiri in Roma*, Agami, Rome (2002). An earlier study, including facsimiles of many original documents, is by A. Schiavo: *La Meridiana di S. Maria degli Angeli*, Istituto Poligrafico e Zecca dello Stato, Rome (1993). Much information is available on the Basilica's website: *www.santamariadegliangeliroma.it* ; select "La Meridiana". Over the past decade C. Sigismondi has made more useful observations and analysis of the meridian line than anyone else. His reports, almost all in Italian, can most easily be found at *arXiv.org* by searching for "Sigismondi" as author. A recent review is C. Sigismondi: 'Lo Gnomone Clementino Astronomia Meridiana in Basilica', *Gerbertus* [free online journal] 7B, 3–80 (2014).
2. The spot of sunlight is actually an image (upside-down) of the sun. The entire church in effect has become a huge *camera obscura*, or pinhole camera. For example, during a partial solar eclipse in 2006, the solar image observed at Santa Maria degli Angeli had a significant 'lunar bite' taken out of it!
3. The described procedure is only for brief explanatory purposes — it was not that actually used to construct the meridian lines.
4. The date of Easter was (then and now) actually defined relative to an *ecclesiastical* vernal equinox, defined as always 21 March, and an *ecclesiastical* full moon, defined by a certain formula that only approximately tracks the astronomical moon. Despite this, one wanted to check the usefulness of this definition by continually monitoring the actual times of vernal equinoxes.
5. J. Heilbron: *The Sun in the Church: Cathedrals as Solar Observatories.* Harvard University Press, Cambridge, Mass. (1999). In 2012 Heilbron gave the Andrew Somerville Memorial Lecture to the BSS on this topic. The history discussed in the present paper relies heavily on Heilbron. In 1997 Charles Aked gave an earlier Somerville Lecture on meridian lines in general: *BSS Bulletin* **97.3**, 24–8 (July 1997).
6. About 1750 L. Vanvitelli renovated the entire church, including interchanging the original nave and transept.
7. D. Karman: 'Michelangelo's "minimalism" in the design of Santa Maria degli Angeli', *Annali di Architettura*, 20, 141–52 (2008).
8. The size and precise location of the aperture (originally of 2 cm diameter) have unfortunately not remained constant over the years.
9. Observations of bright stars and planets on the meridian were possible to the south through a large window behind the aperture.
10. F. Bianchini: *De Nummo et Gnomone Clementino*, part of Bianchini, *De Kalendario et Cyclo Caesaris ac de Paschali Canone*…, Francisci de Comitibus, Rome (1703).
11. For example, see F. Bianchini: *Astronomicae, ac Geographicae Observationes Selectae*, Ed. E. Manfredi, Romanzini, Verona (1737).
12. Heilbron (ref. 5) nicely describes the history of all the measurements of the obliquity. His tables on pp. 63, 135, 136 and 239 form the basis of Fig. 7.
13. One consequence of the declining value of the obliquity is that the positions on the meridian line of the solstices are slowly approaching each other! Over the past 300 years the summer solstice (southern end) has moved 1.5 cm northwards and the winter solstice has moved 8 cm southwards. A second consequence is that, to the nearest tenth of a degree, the value for ε should actually be quoted today as 23.4°, not 23.5°. The accurate value now is 23.43733° (2015.0), declining 0.000130° per year. Current theory indicates that obliquity oscillates between ~22° and ~24.5°, with a quasi-period of ~40,000 years.
14. Of the 31 other observations, seven are in Catamo and Lucarini (2002) (ref. 1) and ten have been posted online (e.g., YouTube) or published by C. Sigismondi. The handiest way to find the Sigismondi reports with data or images we used is at *arXiv.org* — search for the following article numbers: 1106.2948, 1106.2976, 1109.3558, 1201.0510, 1202.1071, and 1412.6096.
15. We did not make accurate measurements of the *timing* of each solar noon.
16. The spot of light is the gnomonic projection of the solar disc onto the horizontal plane of the floor.
17. At solar noon the sun is at its highest altitude angle of the day with a motion almost exactly east–west.
18. The definition of the edge of each solar image is problematic because the angular sizes of the sun and aperture (as seen from the meridian line) lead to a fuzzy penumbra rather than a sharp cutoff. Since we only needed to determine the *centre* of the solar image (rather than its width), we strove only to be consistent in defining the upper and lower edges for lines AA′ and BB′ in Fig. 9. Because the aperture's angular size is much smaller than that of the sun, the size of the fuzzy penumbra is determined by the former. For detailed discussion see F. King, 'Francesco Bianchini, a study in fuzz (or what John Heilbron didn't tell you)', talk at the 2013 BSS Conference in Edinburgh, *BSS Bulletin* 25(ii), 53 (June 2013).
19. The scale, which can be seen in Fig. 9 at a value of ~55.6 for the solar image centre, equals 100 cot (altitude of sun); one unit = height of aperture / 100 = 20.344 cm.
20. As an example, one-quarter of a numeral's height corresponds to an error of ~2.5′ at the summer solstice, decreasing to ~ 0.4′ at the winter solstice.
21. Notes 1 and 14; private communications (2016–17).
22. P.K. Seidelmann (ed.). *Explanatory Supplement to the Astronomical Almanac*, p. 670, University Science Books, Mill Valley, Cal. (1992, second edition).

23. These values, the results from a greatly improved analysis, replace the preliminary values reported by Sullivan at the 2016 BSS Conference at Liverpool.
24. See ref. 10, pp. 48 and 77.
25. For example, Sigismondi (2014, *arxiv.org* paper number 1412.6096) re-analyzes Bianchini's 1701 winter solstice data.
26. Our method also would in principle allow one to lay out an accurate day-by-day date scale along the line without having to observe on 365 days.
27. See ref. 5, pp. 160–4 and ref.1 (Catamo and Lucarini), pp. 75–81.
28. This precession, which is analogous to the wobbling of the axis of a spinning top, is primarily caused by the gravitational attraction of the moon on the equatorial bulge of the Earth. The NCP does not precess in an exact circle, as often stated, because of the steadily changing value of ε.
29. In the Jubilee Year of 2000 a major restoration of the Santa Maria degli Angeli meridian line and precession ellipses was undertaken; other aspects of the basilica were also substantially improved (for example, a major statue of Galileo was added).
30. We do not know which precession equations Bianchini used to calculate the changing NPD of Polaris 800 years into his future. It is unlikely that his calculated values were accurate enough to pin down the minimum NPD of Polaris to a single particular year. He probably found the minimum to be near 2100 and then happily adopted exactly 2100 because it had three desirable properties: a Jubilee year, a century year, and an even number of centuries from a starting point of 1700, only requiring a two-year shift from the construction date of the meridian line. (It turns out that modern calculations in fact predict a minimum in March 2100!)
31. The ratio of minor to major axis for each ellipse is simply sin (latitude) = 0.668.
32. Sigismondi (ref. 1, pp. 61–2) has in addition determined that the major axes of the ellipses are not aligned to true north, but are 3.0′ east of north.
33. The primary contributors were C. Sigismondi (ref. 14) and M. Catamo and C. Lucarini (ref. 1). Others whose names can be identified from the Internet are: N. Bruni, J. Giesen, N. Heckenberg, C. Muccini, R. Num, V. Reijs, and M. Rocco.

Left to right:
Jennifer Look has now received her bachelor's degree in Biology and works for Pure Engineering in California. **Guadalupe Tovar** and **Mallory Thorp** have received their bachelor's degrees in Physics and Astronomy, and will be doing postgraduate studies in Astronomy at the University of Washington (UW) and the University of Victoria (Canada), respectively. **Woody Sullivan** has just retired from the UW faculty after teaching and researching for 40 years in the fields of Astronomy, Astrobiology, and History of Science. He has designed many sundials in the Seattle region, further afield in the USA, on the planet Mars, and tattooed on his wrist.

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